§1.3.1

$\boxed{\text{問題A}}$ $S_1 = 1\times 1 = 1$

$\boxed{\text{II}}$ (1) $S_2 = \frac{1}{2}\times\left(\frac{1}{2}\right)^2 + \frac{1}{2}\times 1^2 = \frac{1}{2}\times\left\{\frac{1}{2^2}+\left(\frac{2}{2}\right)^2\right\} = \frac{1}{2}\times\frac{1+4}{4} = \frac{5}{8} = 0.625$

(2) $S_3 = \frac{1}{3}\times\left(\frac{1}{3}\right)^2 + \frac{1}{3}\times\left(\frac{2}{3}\right)^2 + \frac{1}{3}\left(\frac{3}{3}\right)^2 = \frac{1}{3}\times\left\{\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{3}{3}\right)^2\right\} = \frac{1}{3}\times\frac{1+4+9}{3^2} = \frac{14}{27} = 0.519$

(3) $S_4 = \frac{1}{4}\times\left(\frac{1}{4}\right)^2 + \frac{1}{4}\times\left(\frac{2}{4}\right)^2 + \frac{1}{4}\times\left(\frac{3}{4}\right)^2 + \frac{1}{4}\times\left(\frac{4}{4}\right)^2 = \frac{1}{4}\times\left\{\left(\frac{1}{4}\right)^2+\left(\frac{2}{4}\right)^2+\left(\frac{3}{4}\right)^2+\left(\frac{4}{4}\right)^2\right\}$

$= \frac{1}{4}\times\frac{1+4+9+16}{4^2} = \frac{30}{64} = 0.469$

(4)

$S_n = \frac{1}{n}\times\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\cdots+\left(\frac{n}{n}\right)^2\right\}$

$= \frac{1}{n}\times\frac{1^2+2^2+\cdots+n^2}{n^2} = \frac{1}{n^3}\times\frac{n(n+1)(2n+1)}{6} = \frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

(5)

$S_\infty = \int_0^1 x^2\,dx = \lim_{n\to\infty}\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{1}{3}$