§ 2.1.2

問題A

① (1) $f(x) = \dfrac{1}{x^2} = x^{-2}$ $f'(x) = -2x^{-3} = -\dfrac{2}{x^3}$

(2) $f(x) = \dfrac{1}{x^3} = x^{-3}$ $f'(x) = -3x^{-4} = -\dfrac{3}{x^4}$

(3) $f(x) = (x^3 - x + 1)(x^2 + x + 1)$ $f'(x) = (3x^2 - 1)(x^2 + x + 1) + (x^3 - x + 1)(2x + 1) = 5x^4 + 4x^3$

(4) $f(x) = \dfrac{3x+2}{x-2}$ $f'(x) = \dfrac{3(x-2) - (3x+2)}{(x-2)^2} = \dfrac{3x - 6 - 3x - 2}{(x-2)^2} = \dfrac{-8}{(x-2)^2}$

② (1) $\displaystyle\int x^{-2}\,dx = -x^{-1} + C = -\dfrac{1}{x} + C$

(2) $\displaystyle\int x^{-3}\,dx = -\dfrac{1}{2}x^{-2} + C = -\dfrac{1}{2x^2} + C$

③ (1) $\displaystyle\int_1^3 \dfrac{1}{x^2}\,dx = \int_1^3 x^{-2}\,dx = \left. -x^{-1} \right|_1^3 = -\dfrac{1}{3} + \dfrac{1}{1} = \dfrac{2}{3}$
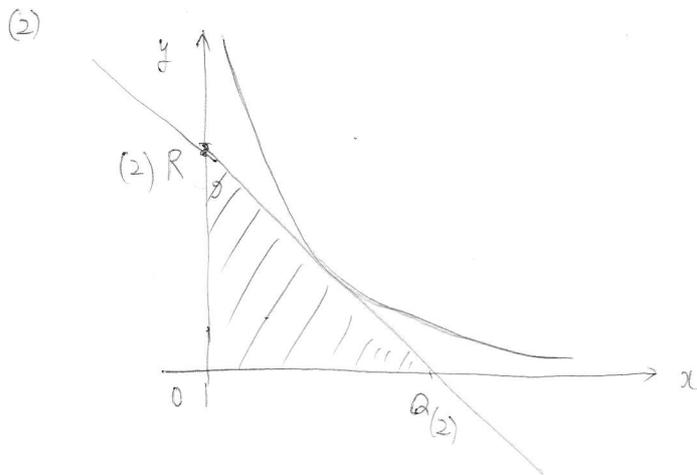
(2) $\displaystyle\int_1^3 \dfrac{1}{x^3}\,dx = \left[-\dfrac{1}{2x^2} \right]_1^3 = -\dfrac{1}{18} + \dfrac{1}{2} = \dfrac{-1+9}{18} = \dfrac{8}{18} = \dfrac{4}{9}$

問題B

[1] (1) $f(x) = \frac{1}{x} = x^{-1}$, $f'(x) = -x^{-2} = -\frac{1}{x^2}$, $f'(1) = -1$

接線の方程式

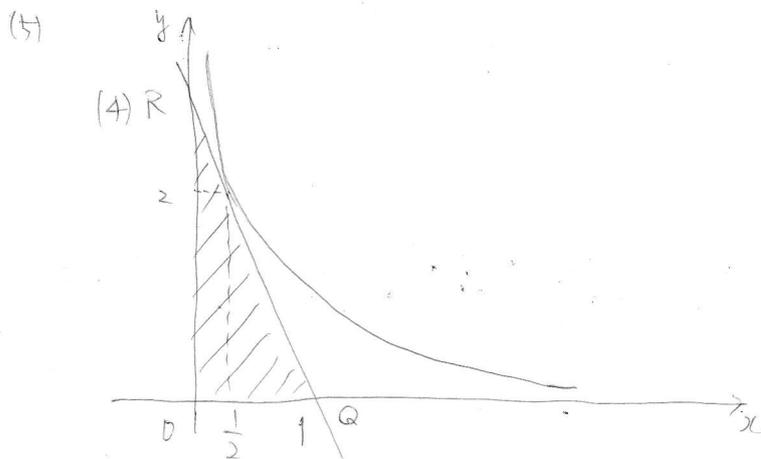$$y - 1 = (-1) \times (x - 1) \qquad \therefore y = -x + 2$$

(2)

(2) R

0   Q(2)   x

(3) $\triangle OQR = \frac{1}{2} \times 2 \times 2 = 2$

(4) 同様に $f'\left(\frac{1}{2}\right) = -4$

$$y - 2 = -4 \times \left(x - \frac{1}{2}\right)$$
$$= -4x + 2 \qquad \therefore y = -4x + 4$$

(5)

(4) R

2

0   $\frac{1}{2}$ 1   Q   x

(6) $\triangle OQR = \frac{1}{2} \times 1 \times 4 = 2$