

2.2 無理関数

問題A

$$\square \text{ (1)} f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \quad f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$$

$$(2) f(x) = x\sqrt{x} = x^{\frac{3}{2}} \quad f'(x) = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x}$$

$$\square \text{ (1)} \int \frac{1}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} + C = 2\sqrt{x} + C$$

$$(2) \int x^{-\frac{1}{3}} dx = \frac{3}{2} x^{\frac{2}{3}} + C$$

$$\square \text{ (1)} \int_0^1 \sqrt{x} dx = \int_0^1 x^{\frac{1}{2}} dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1 = \frac{2}{3}$$

$$(2) \int_1^4 \frac{1}{\sqrt{x}} dx = \int_1^4 x^{-\frac{1}{2}} dx = 2\sqrt{x} \Big|_1^4 = 2\sqrt{4} - 2 = 4 - 2 = 2$$

$$(3) \int_1^8 x^{\frac{1}{3}} dx = \frac{3}{4} x^{\frac{4}{3}} \Big|_1^8 = \frac{3}{4} (2^4 - 1) = \frac{3}{4} \times 15 = \frac{45}{4}$$

$$(4) \int_1^2 \sqrt{x} dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_1^2 = \frac{2}{3} (2\sqrt{2} - 1)$$

$$(5) \int_0^8 \sqrt[3]{x} dx = \int_0^8 x^{\frac{1}{3}} dx = \frac{3}{4} x^{\frac{4}{3}} \Big|_0^8 = \frac{3}{4} \times 2^4 = 12$$

$$(6) \int_1^3 x^{-\frac{3}{2}} dx = -2x^{-\frac{1}{2}} \Big|_1^3 = -\frac{2}{\sqrt{3}} + 2 = 2 - \frac{2}{\sqrt{3}}$$

問題B

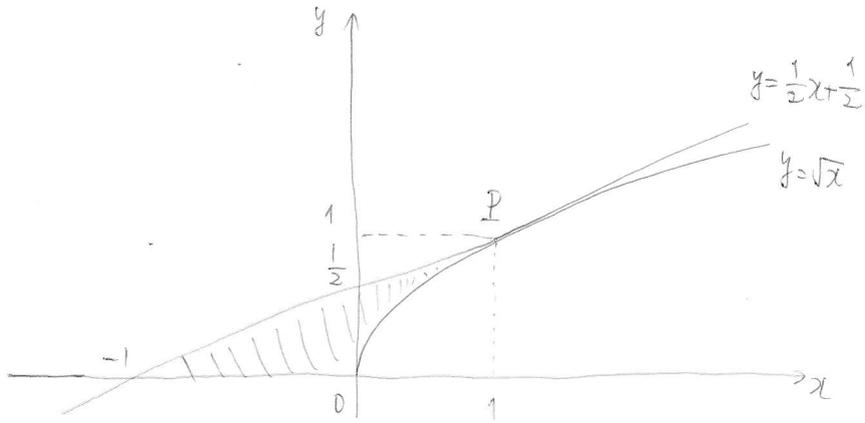
□ (1) $f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$, $f'(1) = \frac{1}{2}$

接線の方程式は

$$y - 1 = \frac{1}{2}x(x-1)$$

$$= \frac{1}{2}x - \frac{1}{2} \quad \therefore y = \frac{1}{2}x + \frac{1}{2}$$

(2)



(3)

$$\frac{1}{2} \times 1 \times 2 - \int_0^1 \sqrt{x} dx$$

$$= 1 - \frac{2}{3}x^{\frac{3}{2}} \Big|_0^1$$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$