§2.3.2

問題A

1 (1) $\int (-2x+1)^3 dx = \int t^3 \frac{dt}{-2} = -\frac{1}{2}\frac{t^4}{4} + C = -\frac{1}{8}(-2x+1)^4 + C$

$t = -2x+1$
$dt = -2dx$

(2) $\int \frac{1}{(9x+8)^4} dx = \int \frac{1}{t^4}\frac{dt}{9} = \frac{1}{9}\int t^{-4} dt = -\frac{1}{27}t^{-3} + C = -\frac{1}{27(9x+8)^3} + C$

$t = 9x+8$
$dt = 9dx$

(3) $\int \sqrt{2x-6}\, dx = \int t^{\frac{1}{2}} \frac{dt}{2} = \frac{1}{2}\frac{2}{3}t^{\frac{3}{2}} + C = \frac{1}{3}(2x-6)^{\frac{3}{2}} + C$

$t = 2x-6$
$dt = 2dx$

(4) $\int x\sqrt{1-x}\, dx = \int (1-t)t^{\frac{1}{2}} \frac{dt}{-1} = -\int \left(t^{\frac{1}{2}} - t^{\frac{3}{2}}\right) dt = -\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + C$

$= -\frac{2}{3}(1-x)^{\frac{3}{2}} + \frac{2}{5}(1-x)^{\frac{5}{2}} + C$

$t = 1-x$
$dt = -dx$

2 (1) $\int_0^1 x(1-x)^5 dx = \int_1^0 (1-t)t^5(-dt) = -\int_1^0 (t^5 - t^6) dt = -\left[\frac{t^6}{6} - \frac{t^7}{7}\right]_1^0 = \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$

$t = 1-x$
$dt = -dx$

x	$0 \to 1$
t	$1 \to 0$

(2) $\int_0^3 (x^2+x)^2(2x+1) dx = \int_0^{12} t^2 dt = \left.\frac{t^3}{3}\right|_0^{12} = \frac{12^3}{3} = 576$

$t = x^2+x$
$dt = (2x+1)dx$

x	$0 \to 3$
t	$0 \to 12$