§3.1.2

問題A

1 (1) $f'(x) = \dfrac{e^x - e^{-x}}{2} = \sinh x$

(2) $f'(x) = \dfrac{e^x + e^{-x}}{2} = \cosh x$

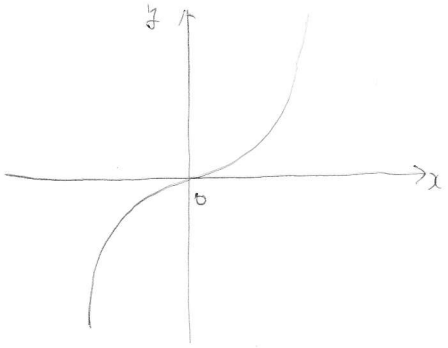
(3) $f'(x) = \dfrac{(e^x+e^{-x})^2 - (e^x-e^{-x})^2}{(e^x+e^{-x})^2} = \dfrac{e^{2x}+1+e^{-2x}-e^{2x}+1-e^{-2x}}{(e^x+e^{-x})^2} = \dfrac{2}{(e^x+e^{-x})^2}$

2 (1) $f(x) = \dfrac{e^x - e^{-x}}{2}$

$f'(x) = \dfrac{e^x + e^{-x}}{2}$

$f''(x) = \dfrac{e^x - e^{-x}}{2}$

x	$\cdots$	0	$\cdots$
$f'(x)$	$+$	$+$	$+$
$f''(x)$	$-$	0	$+$
$f(x)$	↗	0	↗

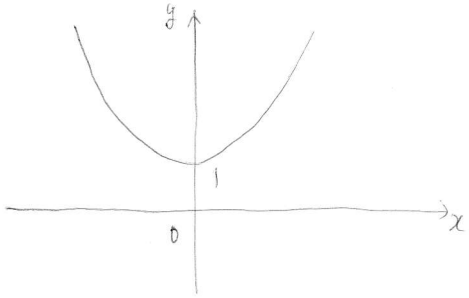


(2) $f(x) = \dfrac{e^x + e^{-x}}{2}$

$f'(x) = \dfrac{e^x - e^{-x}}{2}$

$f''(x) = \dfrac{e^x + e^{-x}}{2}$

x	$\cdots$	0	$\cdots$
$f'(x)$	$-$	0	$+$
$f''(x)$	$+$	$+$	$+$
$f(x)$	↘	1	↗

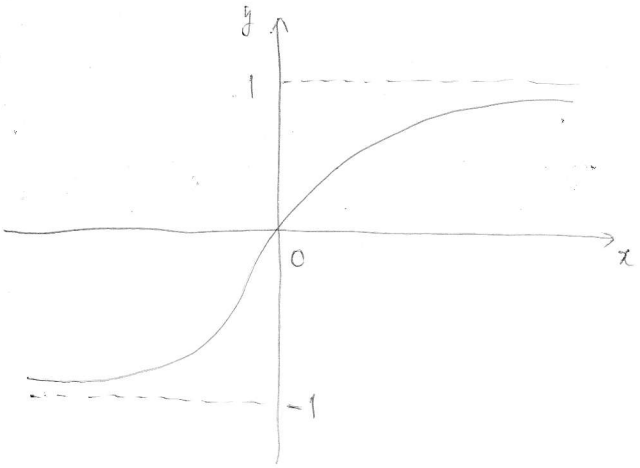


(3) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$f'(x) = \dfrac{2}{(e^x+e^{-x})^2} = 2(e^x+e^{-x})^{-2}$

$f''(x) = -4(e^x+e^{-x})^{-3}(e^x - e^{-x}) = -4\dfrac{e^x - e^{-x}}{(e^x+e^{-x})^3}$

x	$\cdots$	0	$\cdots$
$f'(x)$	$+$	$+$	$+$
$f''(x)$	$+$	0	$-$
$f(x)$	↗	0	↗



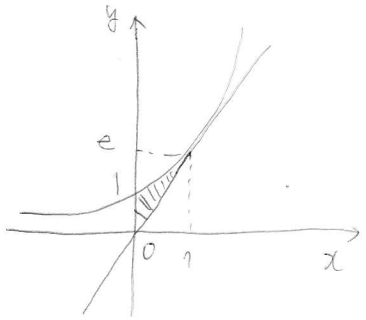
3 (1) $f(1)=e^1$, $f'(x)=e^x$, $f'(1)=e$

接線の方程式

$y-e=e(x-1)$ $\therefore y=ex$

$=ex-e$

(2)



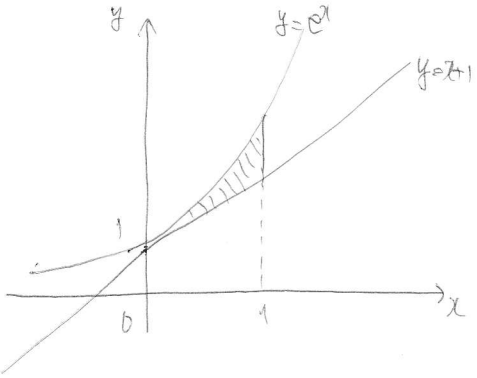
$\int_0^1 e^x dx - \frac{1}{2}\times 1\times e = e^x\Big|_0^1 - \frac{e}{2}$

$= e-1-\frac{e}{2} = \frac{e}{2}-1$

(3)

$y-1=1\times(x-0)$ $\therefore y=x+1$

(4)



$\int_0^1 \{e^x-(x+1)\}dx = e^x-\frac{x^2}{2}-x\Big|_0^1$

$= e-\frac{1}{2}-1-1$

$= e-\frac{5}{2}$

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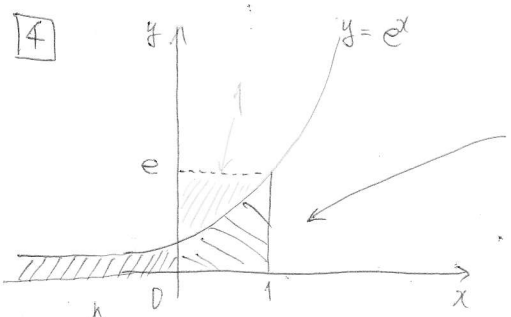


(1) $\int_0^1 e^x dx = e^x\Big|_0^1 = e-1$

(2) $\int_{-\infty}^0 e^x dx = e^x\Big|_{-\infty}^0 = 1$

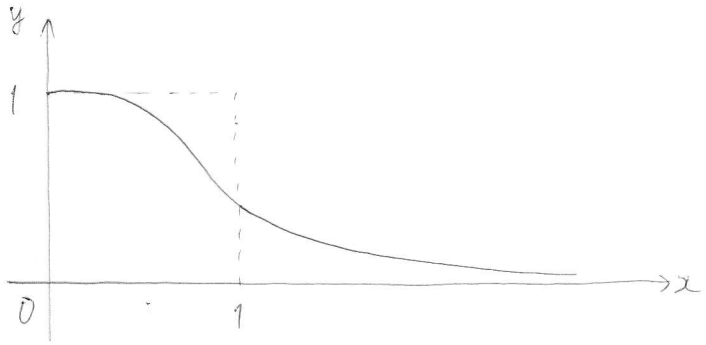
問題B

II (1) $f(x) = e^{-x}(x+1)$

$f'(x) = e^{-x}(-x-1+1) = -x e^{-x}$

$f''(x) = e^{-x}(x-1)$

x	0	⋯	1	⋯
$f'(x)$	0	−	−	−
$f''(x)$	−	−	0	+
$f(x)$	1	↘	$\frac{2}{e}$	↳



(2) $f(x) = e^{-x^2}$

$f'(x) = -2x e^{-x^2}$

$f''(x) = e^{-x^2}(4x^2-2)$

x	⋯	$-\frac{1}{\sqrt{2}}$	⋯	0	⋯	$\frac{1}{\sqrt{2}}$	⋯
$f'(x)$	+	+	+	0	−	−	−
$f''(x)$	+	0	−	−	−	0	+
$f(x)$	↗	$e^{-\frac{1}{2}}$	↱	1	↘	$e^{-\frac{1}{2}}$	↳