§3.1.3

問題A

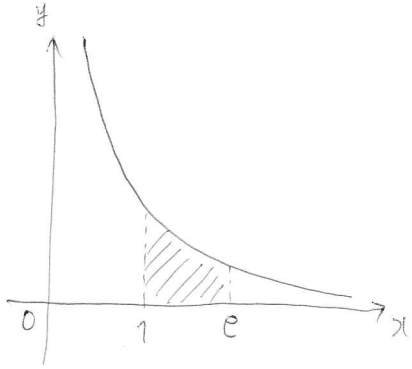
1 (1) $f'(x) = \dfrac{2}{2x} = \dfrac{1}{x}$

(2) $f'(x) = \dfrac{1}{1+x}$

(3) $f'(x) = \dfrac{-1}{1-x}$

(4) $f'(x) = \dfrac{1 + \frac{2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \dfrac{\sqrt{x^2+1}+x}{(x+\sqrt{x^2+1})\sqrt{x^2+1}} = \dfrac{1}{\sqrt{x^2+1}}$

(5) $f'(x) = \dfrac{1 + \frac{2x}{2\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} = \dfrac{\sqrt{x^2-1}+x}{(x+\sqrt{x^2-1})\sqrt{x^2-1}} = \dfrac{1}{\sqrt{x^2-1}}$



2 (1) $S(e) = \displaystyle\int_1^e \frac{dx}{x} = \ln x \Big|_1^e = 1$

(2) $S(2) = \displaystyle\int_1^2 \frac{dx}{x} = \ln x \Big|_1^2 = \ln 2$

$S(3) = \displaystyle\int_1^3 \frac{dx}{x} = \ln x \Big|_1^3 = \ln 3$

(3) $S(2) + S(3) = \ln 2 + \ln 3 = \ln 6 = \displaystyle\int_1^6 \frac{dx}{x}$

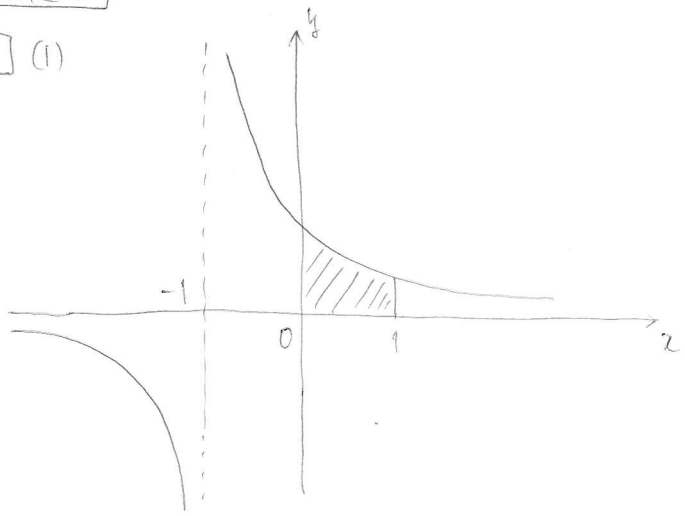
3 (1) $\displaystyle\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\ln(x^2-1) + C$

(2) $\displaystyle\int \frac{e^x}{e^x-1}\,dx = \ln(e^x-1) + C$

(3) $\displaystyle\int \tanh x\,dx = \int \frac{e^x - e^{-x}}{e^x+e^{-x}}\,dx = \ln(e^x+e^{-x}) + C$

~~問題~~ B

1 (1)



(2) $\int_0^1 \frac{1}{x+1}\,dx = \ln(x+1)\Big|_0^1 = \ln 2$

2 (1) $f(x) = e^x - 1 \rightarrow f(0) = 0$
$f'(x) = e^x \rightarrow f'(0) = 1$
$\Big\}$ $y - 0 = 1 \times (x - 0)$ $\therefore y = x$

(2) $f(x) = \frac{e^x - e^{-x}}{2} \rightarrow f(0) = 0$
$f'(x) = \frac{e^x + e^{-x}}{2} \rightarrow f'(0) = 1$
$\Big\}$ $y - 0 = 1 \times (x - 0)$ $\therefore y = x$

(3) $f(x) = \ln(1+x) \rightarrow f(0) = 0$
$f'(0) = \frac{1}{1+x} \rightarrow f'(0) = 1$
$\Big\}$ $y - 0 = 1 \times (x - 0)$ $\therefore y = x$