

3.2 部分積分

問題A

$$\square 1 (1) \int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

$$(2) \int x \ln x dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$(3) \int_1^2 \ln x dx = x \ln x \Big|_1^2 - \int_1^2 x \frac{1}{x} dx = 2 \ln 2 - x \Big|_1^2 = 2 \ln 2 - 1$$

$$(4) \int_0^1 x e^{-x} dx = -x e^{-x} \Big|_0^1 + \int_0^1 e^{-x} dx = -e^{-1} - e^{-x} \Big|_0^1 = -\frac{1}{e} - \frac{1}{e} + 1 = 1 - \frac{2}{e}$$

$$(5) \int_1^2 x \ln x dx = \frac{x^3}{2} \ln x \Big|_1^2 - \int_1^2 \frac{x^3}{2} \frac{1}{x} dx = 2 \ln 2 - \frac{x^2}{4} \Big|_1^2 = 2 \ln 2 - 1 + \frac{1}{4} = 2 \ln 2 - \frac{3}{4}$$

$$\square 2 (1) \int_1^e \ln x dx = x \ln x \Big|_1^e - \int_1^e x \frac{1}{x} dx = e - x \Big|_1^e = e - e + 1 = 1$$

(2) 接点 $(a, \ln a)$ を求め、接線の方程式は、

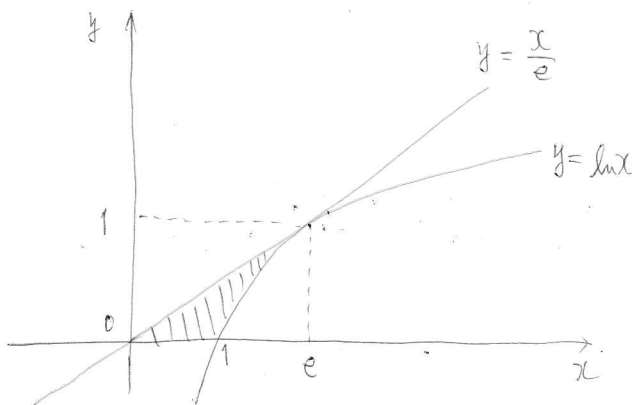
$$y - \ln a = \frac{1}{a}(x - a)$$

この接点 $(e, -1)$ のとき、

$$- \ln a = -1 \quad \therefore a = e$$

したがって、 $y - 1 = \frac{1}{e}(x - e) = \frac{x}{e} - 1$

$$\therefore y = \frac{x}{e}$$



$$(3) \frac{1}{2} \times 1 \times e - \int_1^e \ln x dx = \frac{e}{2} - 1$$