4.3 三角関数の部分積分

問題A

Ⅱ (1) $\int_{-\pi}^{\pi} x\sin x\, dx = -x\cos x\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}(-\cos x)dx = \pi + \pi + \sin x\Big|_{-\pi}^{\pi} = 2\pi$

(2) $\int_{-\pi}^{\pi} x\sin 2x\, dx = -\frac{x}{2}\cos 2x\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\frac{1}{2}\cos 2x\, dx = -\frac{\pi}{2} - \frac{\pi}{2} + \frac{1}{4}\sin 2x\Big|_{-\pi}^{\pi} = -\pi$

(3) $\int_{-\pi}^{\pi} x\cos x\, dx = x\sin x\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\sin x\, dx = +\cos x\Big|_{-\pi}^{\pi} = 0$

(4) $\int_{-\pi}^{\pi} x\cos 2x\, dx = \frac{x}{2}\sin 2x\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{1}{2}\sin 2x\, dx = \frac{1}{4}\cos 2x\Big|_{-\pi}^{\pi} = \frac{1}{4} - \frac{1}{4} = 0$

問題B

Ⅱ (1) $I = \int e^{-x}\sin x\, dx = -e^{-x}\cos x - \int e^{-x}\cos x\, dx$

$$= -e^{-x}\cos x - e^{-x}\sin x - \underbrace{\int e^{-x}\sin x\, dx}_{I}$$

よって

$2I = -e^{-x}\cos x - e^{-x}\sin x + C' \qquad\qquad \therefore I = -\frac{e^{-x}}{2}\left(\cos x + \sin x\right) + C$

(2) $I = \int_{0}^{\frac{\pi}{2}} e^{-x}\cos x\, dx = e^{-x}\sin x\Big|_{0}^{\frac{\pi}{2}} + \int_{0}^{\frac{\pi}{2}} e^{-x}\sin x\, dx$

$$= e^{-\frac{\pi}{2}} + e^{-x}\cos x\Big|_{0}^{\frac{\pi}{2}} - \underbrace{\int_{0}^{\frac{\pi}{2}} e^{-x}\cos x\, dx}_{I} = e^{-\frac{\pi}{2}} + 1$$

よって

$$I = \frac{1 + e^{-\frac{\pi}{2}}}{2}$$